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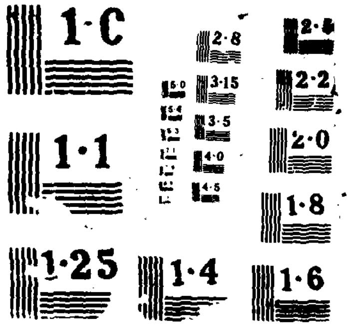
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We are applying our methods to several interesting physical problems, that arise in, e.g., elastic-plastic deformation, combustion, and fluid mechanics.

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ABSTRACT

During the period covered by this report we continued our research on the development and applications of adaptive numerical methods for singularly perturbed initial-boundary value problems for partial differential equations. We continued our analysis of mesh moving schemes, examined local refinement methods, and developed a posteriori error estimation technique for one- and two-dimensional hyperbolic and parabolic problems. We have begun to develop parallel versions of some adaptive procedures.

We are applying our methods to several interesting physical problems, that arise in, e.g., elastic-plastic deformation, combustion, and fluid mechanics.

1. Progress and Status of the Research on Adaptive Numerical Methods.

During the past year we continued the development and analysis of finite difference and finite element methods for rather general systems of parabolic and hyperbolic partial differential equations in one- and two-space dimensions. A list of our publications and manuscripts is given in Section 3 of this proposal and herein we will highlignt some of our key findings.

Papers on the stability of mesh moving schemes [1], one-dimensional mesh moving and refinement techniques [2,4] based on the method of lines, and one- and two-dimensional local refinement schemes [3,5,6] appeared during the last year.

We have extended our adaptive method of lines approach for parabolic systems to two dimensions [7,8,12]. Initially, we calculated piecewise bilinear finite element solutions on an unstructured piecewise rectangular mesh. An error estimate was obtained by adding a piecewise quadratic ''serendipity'' correction to the piecewise bilinear finite element solution. The superconvergence property of finite element methods for parabolic systems was exploited to neglect the error at nodes and increase the computational efficiency of the procedure. The finite element solution and error estimate were integrated in time using software for stiff ordinary differential equations. The error estimate was used to control a spatial refinement procedure whereby elements were added to or deleted from the mesh in regions in high or low error, respectively. We have proven that this error estimate converges to the true error as the mesh is refined and are writing a paper describing our results [13].

One natural extension to these finite element methods is the addition of higher-order approximations to the solution spaces. However, the problem of developing computationally efficient error estimations for these solutions has been quite substantial. Recently, Babuska and Yu obtained some theoretical results that are quite helpful in this regard. They proved that the discretization error of odd-order finite element solutions of linear

elliptic systems is mostly due to jumps in solution derivatives across element boundaries. More significantly, they showed that the majority of the error in even-order approximations occurs in the interior of the finite elements and jumps across element boundaries are negligible. This provides a strong motivation to consider adaptive procedures based on the use of even-order polynomial approximations, since an error estimate could be computed in an element-by-element manner without any assembly. Of course, the higher-order approximations would also converge at a faster rate, which is valuable when high accuracy is needed.

With these observations in mind, we recently added piecewise serendipity functions to the solution spaces of our two-dimensional finite element codes for parabolic problems. Since we have been using serendipity functions to provide error estimates for bilinear solutions, we used a hierarchic approach and easily constructed the serendipity solutions by adding the serendipity error estimate to the bilinear solutions and correcting the nodal values. An error estimate was obtained using a special three-term fifth-order polynomial that vanishes on the edges of each finite element. Due to the hierarchic methodology and local error estimates, the serendipity solutions and fifth-order error estimates required only minor modifications to our dynamic tree data structure. The results provide dramatic improvements in accuracy per unit cost relative to those obtained using bilinear approximations. A paper describing these findings has appeared in the IMACS proceedings [8]. A second paper containing additional material has been accepted for publication in *Numerische Mathematik* [12]. Further investigation is revealing that Babuska and Yu's results might even apply to triangular meshes.

The method of lines approach produces a method with great stability: however, there are many instances where the extra computational effort associated with coupling mesh motion with the solution and error estimate is not necessary. For this reason, we have been considering adaptive finite difference and finite element methods that locally refine the mesh in both space and time. Solutions are initially generated on a course grid for

one time step. The discretization error at the end of the time step is estimated, finer space-time subgrids are added to regions of high error, and the problem is recursively solved again on the finer grids. The process terminates and the integration continues to the next time step when the estimated error at the end of the initial time step satisfies a prescribed tolerance. Our progress on a one-dimensional finite element method that uses uniform rectangular space-time grids appeared in a text on adaptive methods (cf. Bieterman, Flaherty, and Moore [2]). This paper contains some comparisons of our local refinement method with a finite element method of lines due to Bieterman and Babuska. Further developments will be reported in a forthcoming manuscript [14]. We have also developed a local refinement finite volume code for two-dimensional initial-boundary value problems [6,7,11]. In this approach, a course mesh is moved and finer space-time grids are recursively added to the coarse mesh in regions where greater resolutions is needed. The fine grids are properly nested within coarse mesh boundaries; thus, reducing interpolation difficulties at coarse mesh-fine mesh interfaces.

We are extending our adaptive software and methodologies to include enhancements and improvements, such as, higher order polynomial approximations, special upwind approximations for singularly perturbed problems, combined h- and p- version finite element refinement, techniques for non-rectangular two-dimensional regions, and faster linear algebraic (e.g., conjugate gradient and multigrid) techniques for our implicit methods. We have also begun to consider adaptive techniques for there-dimensional problems. This has necessitated the use of parallel and vector computers; thus, we have been testing parallel versions of our refinement, solution, and error estimation algorithms on our Sequent systems.

2. Interactions

Professor Flaherty, Dr. Adjerid, and graduate students supported by this grant lectured and/or visited the following conferences and organizations during the period covered by this report:

S. Adjerid and J.E. Flaherty attended the SIAM National Meeting, Boston, 21-25 July 1986. S. Adjerid presented a paper on "A Local Refinement Finite Element Method for Two-Dimensional Parabolic Systems" and J.E. Flaherty presented a paper in the Minisymposium on Domain Decomposition on "Adaptive Finite Element Methods and Their Relations to Domain Decomposition."

S. Adjerid and J.E. Flaherty attended the conference on "The Impact of Mathematical Analysis on the Solution of Engineering Problems" at the University of Maryland, College Park, 17-19 September 1986. J.E. Flaherty lectured on "Local Refinement Finite Element Methods using Piecewise Bilinear and Cubic Approximations for Parabolic Systems."

J.E. Flaherty attended the Air Force Armament Laboratory Supercomputing Conference, Eglin Air Force Base, 3-5 November 1986. He lectured on "Adaptive Methods for Time-Dependent Partial Differential Equations."

J.E. Flaherty participated in the Computational Sciences Seminar of the United Technologies Research Center, 21 November 1986. He lectured on "Adaptive Methods for Time-Dependent Partial Differential Equations."

J.E. Flaherty lectured on "Adaptive Methods for Time Dependent Partial Differential Equations" at Boeing Computer Services, Bellevue, 12 January 1987.

J.E. Flaherty lectured on "Adaptive Methods for Partial Differential Equations" at MIT on 17 April 1987.

J.E. Flaherty and Mark Shephard attended the Mathematical Aspects of Finite Element Computations and Applications at Brunel University, Uxbridge, 28 April - 1 May 1987. They presented a joint lecture on ''Adaptive Analysis for Automated Finite Element Modeling.''

3. List of Publications and Manuscripts in Preparation.

Publications

1. J.M. Coyle, J.E. Flaherty, and R. Ludwig, "On the Stability of Mesh Equidistribution Strategies for Time-Dependent Partial Differential Equations," *J. Comput. Phys.*, 62 (1986), pp. 26-39.

2. S. Adjerid and J.E. Flaherty, "A Moving Finite Element Method with Error Estimation and Refinement for One-Dimensional Time Dependent Partial Differential Equations," *SIAM J. Numer. Anal.*, 23 (1986), pp. 778-796.

3. M. Bieterman, J.E. Flaherty, and P.K. Moore, "Adaptive Refinement Methods for Non-Linear Parabolic Partial Differential Equations," Chap. 19 in *Accuracy Estimates and Adaptive Refinements in Finite Element Computations*, I. Babuska, O.C. Zienkiewicz, J.R. Gago, and E.R. de A. Olivera, Eds., John Wiley and Sons, Chichester, 1986.

4. S. Adjerid and J.E. Flaherty, "A Moving Mesh Finite Element Method with Local Refinement for Parabolic Partial Differential Equations," *Comp. Meths. Appl. Mech. Engr.*, 56 (1986), pp. 3-26.

5. D.C. Arney and J.E. Flaherty, "A Two-Dimensional Mesh Moving Technique for Time Dependent Partial Differential Equations," *J. Comput. Phys.*, 67 (1986), pp. 124-144.

6. D.C. Arney and J.E. Flaherty, "An Adaptive Method with Mesh Moving and Refinement for Time-Dependent Partial Differential Equations," *Trans. Fourth Army Conf. Appl. Maths. and Comput.*, ARO Report 87-1, U. S. Army Research Office, Research Triangle Park, NC, (1987), pp. 1115-1142.

7. S. Adjerid and J.E. Flaherty, "Adaptive Finite Element Methods for Parabolic Partial

Differential Systems in One and Two Space Dimensions," Trans. Fourth Army Conf. on Applied Maths. and Comput., ARO Report 87-1, U. S. Army Research Office, Research Triangle Park, NC, (1987), pp. 1077-1098.

8. S. Adjerid and J.E. Flaherty, "First- and Second-Order Adaptive Finite Element Methods for Parabolic Systems," in R. Vichnevetsky and R.S. Stepelman, Eds., *Advances in Computer Methods for Partial Differential Equations - VI*, IMACS, New Brunswick, (1987), pp. 472-478.

In Press

9. J.E. Flaherty and T.L. Jackson, "A Discontinuous Finite Element Method for Hyperbolic Systems of Conservation Laws," Technical Report 85-33, Department of Computer Science, Rensselaer Polytechnic Institute, Sept. 1985. Also *SIAM J. Sci. Stat. Comput.*, submitted for publication Sept. 1985.

10. S. Adjerid and J.E. Flaherty, "A Local Refinement Finite Element Method for Two-Dimensional Parabolic Systems," Tech. Rep. No. 86-7, Department of Computer Science, Rensselaer Polytechnic Institute, Troy, May 1986. Also submitted for publication in *SIAM J. Sci. Stat. Comput.*, accepted for publication, April 1987.

11. D.C. Arney and J.E. Flaherty, "An Adaptive Local Mesh Refinement Method for Time-Dependent Partial Differential Equations," Technical Report 86-10, Department of Computer Science, Rensselaer Polytechnic Institute, July 1986. Also *SIAM J. Sci. Stat. Comput.*, submitted for publication, May 1986.

12. S. Adjerid and J.E. Flaherty, "Second Order Finite Element Approximations and A Posteriori Error Estimation," Technical Report 87-1, Department of Computer Science, Rensselaer Polytechnic Institute, January 1987. Also, *Numer. Math.*, accepted for publication, February 1987.

In Preparation

13. S. Adjerid and J.E. Flaherty, ''Local Refinement Finite Element Methods on Stationary and Moving Meshes for Parabolic Systems,'' in preparation for *SIAM J. Numer. Anal.*.

14. P.K. Moore and J.E. Flaherty, ''A Local Refinement Finite Element Method for One-Dimensional Parabolic Systems, in preparation for *J. Comput. Phys.*.

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